Five Methods for MC Speedy Solving

As shown in the following table, students have to attempt 45 multiple-choice questions within 1 hour 15 minutes in Paper 2 of HKDSE Examination. Although the duration and the number of questions are both less than that of HKCEE, the average time for attempting each question remains the same. Therefore, students need to master different answering skills and make use of quick and accurate methods. Here are 5 most common methods for solving multiple-choice questions:

1. Direct Calculation Method
2. Elimination Method
3. Trial Method
4. Special Value Method
5. Graphical Method

Paper 2	HKDSE Examination (Compulsory Part)	HKCEE
Weighting	35%	40%
Number of Questions	Section A: 30 Section B: 15	Section A: 36 Section B: 18
Duration	1 hour 15 minutes	1 hour 30 minutes
Average Time for Attempting Each Question	$1\frac{2}{3}$ minutes	$1\frac{2}{3}$ minutes

① Direct Calculation Method

Use the given formulas, theorems and rules to calculate the answer directly.

Hints: This method is often used in questions involving percentages, mensuration and deductive geometry.

Example Nancy sold two skirts for \$198 each. She gained 10% on one and lost 10% on the other. After the two transactions, Nancy

A. lost \$4.
B. gained \$10.
C. gained \$22.
D. had no gain and no loss.

Adapted from HKDSE Sample Paper 09 Q10

Solution **Get the answer directly by using the following formulas:**

$$\text{Profit Percentage} = \frac{\text{Selling Price} - \text{Cost}}{\text{Cost}} \times 100\% \text{ and Loss Percentage} = \frac{\text{Selling Price} - \text{Cost}}{\text{Cost}} \times 100\%$$

Let \$$x$ be the cost of the skirt that made profit, \$$y$ be the cost of the skirt that made loss.

$$10\% = \frac{198 - x}{x} \times 100\% \quad \text{and} \quad 10\% = \frac{y - 198}{y} \times 100\%$$

$$0.1x = 198 - x \quad \text{and} \quad 0.1y = y - 198$$

$$1.1x = 198 \quad \text{and} \quad -0.9y = -198$$

$$x = 180 \quad \text{and} \quad y = 220$$

Total cost = 180 + 220 = \$400; total income = 198 × 2 = \$396

∵ Total cost > Total income

∴ Nancy's lost after the two transactions = 400 − 396 = \$4

∴ Option A is the correct answer.

② Elimination Method

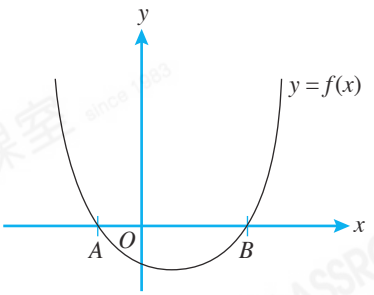
First, determine the options that must not be the correct answer. Then, concentrate on the remaining options.

Hints: (i) This method is especially useful when tackling the problems that examining concepts and determination of statements.

(ii) Eliminate the options carefully. Do not eliminate by intuitive.

(iii) Observe the combination of the options first when answering the questions. In some cases, determination of some statements is already enough to get the correct answer.

Example In the figure, the quadratic graph of $y = f(x)$ intersects the x-axis at $A(-1, 0)$ and $B(5, 0)$. Which of the following are true?



I. The roots of the equation $f(x) = 0$ are -1 and 5.

II. The equation of the axis of symmetry of the quadratic graph of $y = f(x)$ is $x = 3$.

III. The solution of the inequality $f(x) > 0$ is $x < -1$ or $x > 5$.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Adapted from HKDSE Sample Paper 09 Q8

Solution **Observing the four options, each statement appears three times. Therefore, we can get the correct answer immediately if we can find an incorrect statement. If all the statements are correct, then option D is the correct answer.**

I. ∵ The x-intercepts of the graph are the roots of the equation

∴ The roots of the equation are -1 and 5

∴ I is correct.

II. Consider that the mid-point of AB on the axis of symmetry.

The mid-point of $AB = (\frac{-1+5}{2}, 0)$

$= (2, 0)$

∴ The equation of the axis of symmetry of the graph is $x = 2$

∴ II is incorrect.

∵ Only option B does not contain II

∴ Option B is the correct answer.

III. In the range of $x < -1$ and $x > 5$ of the graph,

the graph of $y = f(x)$ is on top of the x-axis,

i.e. when $f(x) > 0$, $x < -1$ or $x > 5$

∴ III is correct.

③ Trial Method

Test the options one by one to find the correct answer.

Hints: (i) Questions that can be done from the answer can use Trial Method.
(ii) This method is often used with Elimination Method.

Example The coordinates of the points A and B are $(4\ ,7)$ and $(8\ ,1)$ respectively. If P is a point lying on the straight line $y = x - 1$ such that $AP = PB$, then the coordinates of P are

A. $(3\ ,2)$.
B. $(3\ ,4)$.
C. $(5\ ,4)$.
D. $(5\ ,6)$.

Adapted from HKCEE09 Q31

Solution **Using Direct Calculation Method or Elimination Method + Trial Method**

1. Using Direct Calculation Method:

Let $(x_1\ , y_1)$ be the coordinates of P.

$\because$ P is a point lying on the straight line $y = x - 1$

$\therefore\ y_1 = x_1 - 1$ ①

$\because\ AP = PB$

$\therefore\ \sqrt{(x_1-4)^2+(y_1-7)^2} = \sqrt{(x_1-8)^2+(y_1-1)^2}$

$x_1^2 - 8x_1 + 16 + y_1^2 - 14y_1 + 49 = x_1^2 - 16x_1 + 64 + y_1^2 - 2y_1 + 1$

$8x_1 - 12y_1 = 0$ ②

Substitute ① into ②,

$8x_1 - 12(x_1 - 1) = 0$

$x_1 = 3$

Substitute $x_1 = 3$ into ①,

$y_1 = 3 - 1 = 2$

$\therefore$ The coordinates of P are $(3\ ,2)$.

$\therefore$ Option A is the correct answer.

2. Using Elimination Method + Trial Method:

It is given that P is a point lying on the straight line $y = x - 1$. Observing the four options, only A $(3\ ,2)$ and C $(5\ ,4)$ fulfil the requirement. Therefore, options B and D can be eliminated. Then, test either option A or C randomly.

A. Consider the distance between $P\ (3\ ,2)$ and A and the distance between $P\ (3\ ,2)$ and B,

$AP = \sqrt{(4-3)^2+(7-2)^2} = \sqrt{26}$

$PB = \sqrt{(8-3)^2+(1-2)^2} = \sqrt{26}$

$\therefore\ AP = PB$

$\therefore$ Option A is the correct answer.

In the above example, students need to spend more time when using Direct Calculation Method. Moreover, it involves complicated algebraic calculations and students make mistakes more easily. By using Elimination Method and Trial Method, we can eliminate two options first. Then, we only need to test one of the remaining options to get the correct answer. Students do not need to deal with tedious algebraic expressions.

④ Special Value Method

For faster calculation, substitute simple values into the variables in the question.

Hints: (i) Substitute values that do not violate the requirements of the question, and do not make the operations more complicated.

(ii) This method is often used in questions involving algebra or abstract concepts. When students face difficult questions, try substituting some values to calculate.

Example Let p and q be constants. If $-2x^2 + p(x-5) + q \equiv -2(x+1)(x-3)$, then $q =$

A. -26.

B. 4.

C. 10.

D. 26.

Adapted from HKDSE Sample Paper 09 Q4

Solution **In this question, it is more complicated if we expand both sides of the identity, and then compare the coefficient of x and the constant term. Consider when $x = 5$, the p on the L.H.S. of the identity will be eliminated, therefore we substitute $x = 5$ into the identity.**

Substitute $x = 5$ into the identity,

L.H.S. $= -2(5)^2 + p(5-5) + q = -50 + q$

R.H.S. $= -2(5+1)(5-3) = -24$

$\therefore -50 + q = -24$

$q = 26$

$\therefore$ Option D is the correct answer.

Example If the length and the width of a rectangle are increased by 10% and x% respectively so that its area is increased by 32%, then $x =$

A. 10.

B. 12.

C. 20.

D. 22.

Adapted from HKDSE Sample Paper 09 Q12

Solution **Using Special Value Method + Direct Calculation Method**

Suppose that the length and the width of the rectangle are 200 cm and 100 cm respectively.

The area of the new rectangle $= (200 \times 100) \times (1 + 32\%) = 26\,400$ cm^2

The length of the new rectangle $= 200 \times (1 + 10\%) = 220$ cm

The width of the new rectangle $= 100 \times (1 + x\%) = (100 + x)$ cm

Therefore, the area of the new rectangle $= 220 \times (100 + x) = (22\,000 + 220x)$ cm^2

$22\,000 + 220x = 26\,400$

$220x = 4\,400$

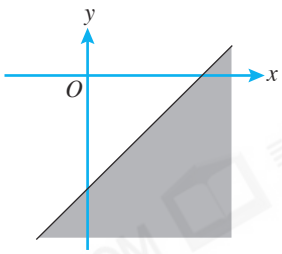
$x = 20$

$\therefore$ Option C is the correct answer.

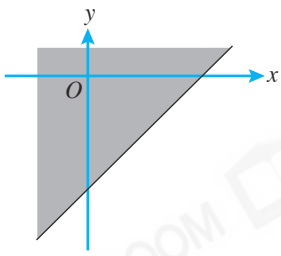
In the above example, students may use different variables to represent the length and the width, and then find the value of x. However, it is more complicated.

Example Which of the following shaded regions may represent the solution of $x + y + 5 \leq 0$?

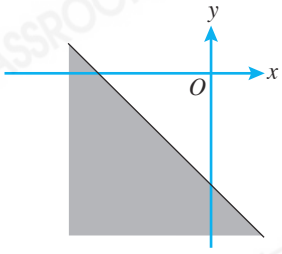
A.



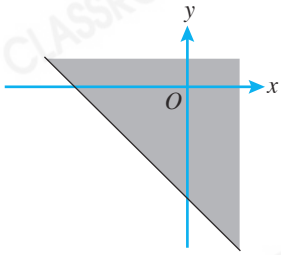
B.



C.



D.



Adapted from HKCEE09 Q44

Solution **Using Elimination Method + Special Value Method, we can find the answer more quickly.**

The slope of the straight line $x + y + 5 = 0$ is -1.
Observing the four options, only the slopes of the straight lines of the graphs of Options C and D are negative. Therefore, Options A and B can be eliminated.
Substitute $(0\ ,0)$ into $x + y + 5 \leq 0$.
$\because\ 0 + 0 + 5 = 5 > 0$
$\therefore$ $(0\ ,0)$ is not in the shaded region
$\therefore$ Option C is the correct answer.

⑤ Graphical Method

Sketch diagrams according to the question and find the hidden information.

Hints: For problems without figure, such as graphs of functions and trigonometry, sketching diagrams can be more effective to get the key points of the question and build up related expressions.

Example A and B are two points on a map. If the bearing of A from B is 080°, then the bearing of B from A is

A. 080°.
B. 100°.
C. 260°.
D. 280°.

Adapted from HKCEE07 Q15

Solution **The question does not provide figure. Some students mistakenly subtract 80° from 360° to get the answer. However, we can get the correct answer easily if we sketch the following diagram.**

$\alpha + 80° = 180°$ (adj. ∠s on st. line)
$\alpha = 100°$
$\beta = \alpha = 100°$ (alt. ∠s, // lines)
$\theta + 100° = 360°$ (∠s at a point)
$\theta = 260°$
$\therefore$ The bearing of A from B is 260°.
$\therefore$ Option C is the correct answer.